A NEW WIDEBAND DISCRIMINATOR

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ABSTRACT. The present note describes a discriminator, employing transmission lines, designed to have linear response over a wide range of frequency deviation as is required in wideband FM work. It has been shown that linearity of response can be obtained without difficulty up to a fractional frequency deviation as high as 50 per cent.

INTRODUCTION

Conventional tuned circuit discriminators give linear response for frequency deviations which are at best small fractions of the central carrier frequency and are hence unsuitable for wideband FM work. In this paper a new type of discriminator has been described which can be arranged to give linear response over a range even wider than is necessary for wideband FM work in current use. Theoretically, the output of the discriminator is shown to be linear to within 5% up to a fractional frequency deviation as high as 70%. In a practical arrangement, however the range of linearity is reduced to about 50%, but even this range is considerably higher than what can be approached with ordinary tuned circuit discriminators. The discriminator employs physical or artificial transmission lines for the generation of sinusoidal frequency functions which are then combined to give the familiar '*S*' characteristic.

Circuit arrangements for different frequency regions are described.

PRINCIPLE OF THE CIRCUITS

Let us consider the circuit of figure 1(a). The voltage V_{BC} across BC, is the sum of the voltage drops V_{DC} and V_{BD}. Each of these voltages is developed across resistance R_0 which is the characteristic impedance of the transmission line of delay δ. Since the travelling current wave at B appears δ secs. after the application of the input at A, the voltage V_{BC} can be written as

$$V_{BC}(t) = R_0[I(t)+I(t-\delta)],$$

and operationally as

$$V_{BC}(p) = R_0(1+e^{-p\delta}) \,.\, I(p)$$

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Hence for a sinusoidal current $Ie^{j\omega t}$

$$V_{BC}(j\omega) = R_0[1+e^{-j\omega\delta}]I = 2R_0 I \cos\frac{\omega\delta}{2} \cdot e^{j\frac{\omega\delta}{2}},$$

or

$$|V_{BC}| = 2R_0 I \cos\frac{\omega\delta}{2} \qquad \dots \quad (1)$$

The input impedance Z_{AC} equals $2R_0$; hence the input voltage is $2R_0 I$. The voltage across AB is the difference between the input voltage V_{AC} and the voltage V_{BC}. Hence

$$V_{AB}(j\omega) = (1-e^{-j\omega\delta})R_0 I = 2R_0 I \sin\frac{\omega\delta}{2} \cdot e^{j\frac{\omega\delta}{2}+j\pi/2}$$

or

$$|V_{AB}| = 2R_0 I \sin\frac{\omega\delta}{2} \qquad (2)$$

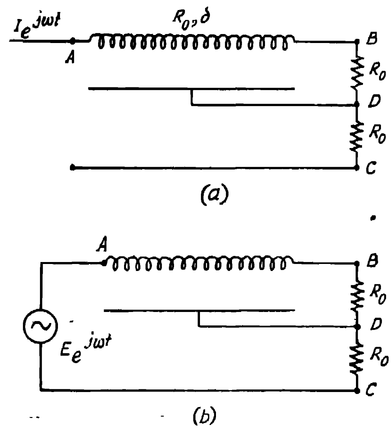


Fig. 1. Schematic diagram of a transmission line arrangement for producing sinusoidal voltage output.

If the circuit be voltage fed as in figure 1(b), the voltage drops are given by

$$|V_{BC}| = E \cos\frac{\omega\delta}{2} \qquad \dots \quad (1.1)$$

$$|V_{AB}| = E \sin\frac{\omega\delta}{2} \qquad \dots \quad (2.1)$$

Now the voltages V_{BC} and V_{AB} which are sinusoidal functions of frequency ω can be combined to give a discriminator characteristic. Thus, from elementary trigonometry, the difference of two voltages of the form $A_1 \sin \frac{\omega\delta}{2}$ and $A_2 \cos \frac{\omega\delta}{2}$ can be written as:

$$A_1 \sin \frac{\omega\delta}{2} - A_2 \cos \frac{\omega\delta}{2} = A_0 \sin \left(\frac{\omega\delta}{2} - \phi \right) \quad \ldots \quad (3)$$

$$= V_0, \text{ say,}$$

where $A_0 = \sqrt{A_1^2 + A_2^2}$ and $\phi = \tan^{-1} \frac{A_2}{A_1}$;

It is seen that V_0 is zero at a frequency defined by

$$\omega_0 = \frac{\tan^{-1} \frac{A_2}{A_1}}{\delta/2}. \quad (4)$$

For any other frequency

$$\omega = \omega_0 + \Delta\omega,$$

$$V_0 = A_0 \sin \left(\Delta\omega \, . \, \frac{\delta}{2} \right) \quad \ldots \quad (5)$$

Now $\sin \theta/\theta$ is nearly equal to unity for small values of θ. In fact, up to $\theta = \pi/6$ radians, $\sin \theta/\theta$ does not depart from unity by more than 5%. Hence it follows that the difference voltage V_0 will be linearly related to $\Delta\omega$, for values of $\Delta\omega \, . \, \frac{\delta}{2}$ up to $\pi/6$ radians, the departure not exceeding 5%.

Now, for $\Delta\omega \, . \, \frac{\delta}{2} = \pi/6$, $\frac{\Delta\omega}{\omega_0} = \frac{\pi/6}{\tan^{-1} \frac{A_2}{A_1}}$, from Eq. (4).

If $A_1 = A_2$, $\frac{\Delta\omega}{\omega_0} = \frac{2}{3} . = 66.6\%$

It is thus clear that by combining differentially the two voltages V_{AB} and V_{BC}, as given by eqns. (1) and (2) or by eqns. (1.1) and (1.2), a discriminator may be arranged. The output of such a discriminator, working with FM waves of carrier frequency ω_0 with a frequency deviation $\Delta\omega$, will obviously be linear (to within 5%) for fractional frequency deviations as large as 67%.

It should be noted that this limiting figure is obtained with the constraint that $A_1 = A_2$. If, however, this constraint is removed and the magnitudes of

A_1 and A_2 are suitably adjusted, the linearity can be extended even up to 100% frequency deviation.

It may be noted that the component functions $\cos \frac{\omega\delta}{2}$ and $\sin \frac{\omega\delta}{2}$ themselves provide discriminator characteristics around the centre frequencies $\omega = (2K+1)\frac{\pi}{\delta}$ and $\omega = \frac{2K\pi}{\delta}$ respectively while the combined characteristics $\sin\left(\frac{\omega\delta}{2} - \phi,\right)$ as seen above, is centered round $\omega = \frac{(\phi + K\pi)}{\delta/2}$ where K is any integer. Thus there will be harmonic response. It is therefore necessary to provide the required attenuation at all but the desired mode. The separation between successive "modes" in all the three cases is $2\pi/\delta$, but the ratio between the centre frequencies corresponding to the first two modes can be made higher in the combined characteristics than in the other two cases. What is more important is that the S/N ratio is better in the balanced form, and the maximum fractional frequency deviation is the largest.

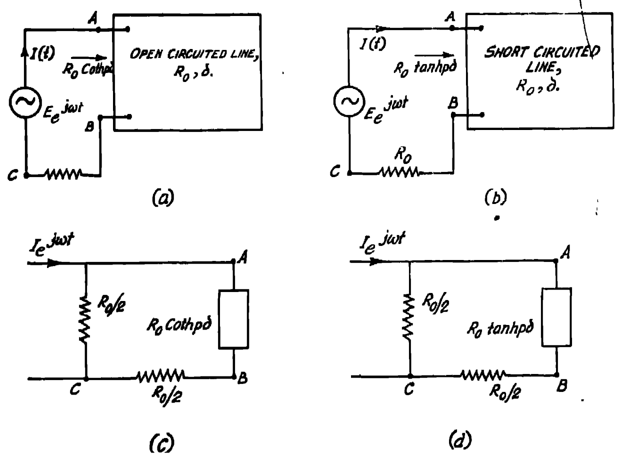


Fig. 2. Arrangements employing a single transmission line for generating sinusoidal magnitude functions.

Other circuit arrangements for realising sinusoidal magnitude function and hence discriminator characteristics are shown in figure 2. Thesea rrangements utilise reflections at the end of the transmission line and are discussed below.

Figure 2(*a*) :

$$I(t) = \frac{E(t)}{2R_0} - \frac{E(t-2\delta)}{2R_0}$$

and $$I(j\omega) = \frac{E}{R_0} \sin \omega\delta \,.\, e^{j(\omega\delta+\pi/2)}.$$

Hence the voltage $$V_{BC} = j\, E \sin \omega\delta\, e^{j(\omega\delta+\pi/2)}$$

and $$V_{AB} = E \cos \omega\delta \,.\, e^{j\omega\delta}\ ,$$

$$V_0 = |V_{BC}| - |V_{AB}| = \sqrt{2}E \sin(\omega\delta - \pi/4) = \sqrt{2}E \sin \Delta\omega \,.\, \delta,$$

for $$\omega = \omega_0 + \Delta\omega, \text{ where } \omega_0 = \pi/4\delta.$$

Figure 2(*b*) :

$V_{AB} = jE \sin \omega\delta \,.\, e^{j\omega\delta}$ and $V_{BC} = E \cos \omega\delta \,.\, e^{j\omega\delta/2}$ and hence
$V_0 = |V_{AB}| - |V_{BC}| = \sqrt{2}\, E \sin(\omega\delta - \pi/4)$.

Both the systems 2(*a*) and 2(*b*) are fed from a constant voltage generator.

Figure 2(*c*) :

The current in the branch *CBA* is

$$I_B = \frac{R_0/2}{R_0 + R_0 \tanh p\delta} \,.\, I = \frac{I}{2} \,.\, \frac{1}{1+\tanh p\delta}$$

Here the voltage $$|V_{AB}| = \frac{I.R_0}{2} \sin \omega\delta$$

and $$|V_{BC}| = \frac{IR_0}{4} \cos \omega\delta,$$

Hence, $$V_0 = |V_{AB}| - |V_{BC}| = \frac{IR_0}{4} \,.\, \sqrt{5}.\, \sin(\omega\delta - \tan^{-1} 0.5)$$

For $$\omega = \omega_0 + \Delta\omega, \text{ where } \omega_0 = \tan^{-1} \frac{0.5}{\delta},$$

$$V_0 = \frac{IR_0}{4} \,.\, \sqrt{5} \quad \sin(\Delta\omega \,.\, \delta).$$

Figure 2(*d*) :

$$|V_{AB}| = \frac{IR_0}{2} \cos \omega\delta,$$

$$|V_{BC}| = \frac{IR_0}{4} \sin \omega\delta,$$

and $$V_0 = |V_{BC}| - |V_{AB}| = \frac{IR_0}{4} \,.\, \sqrt{5} \sin(\delta\omega - \tan^{-1} 0\ 2).$$

In figure 3, are shown arrangements where the sinusoidal functions are generated by means of two lines, one open and the other short circuited.

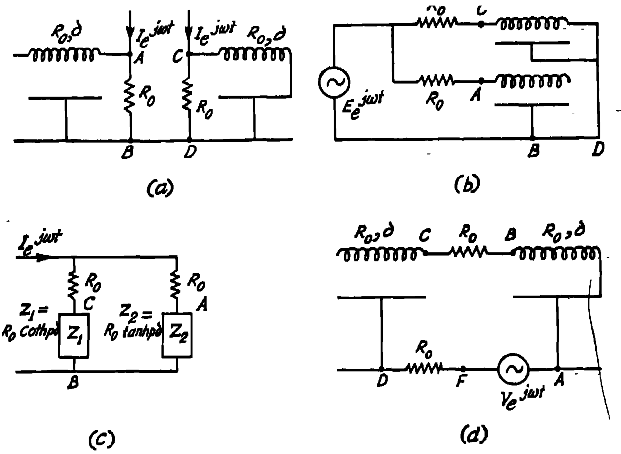


Fig. 3. Arrangements employing two transmission lines for generating sinusoidal magnitude functions.

Figure 3(a) :

$$|V_{AB}| = IR_0 \sin \omega\delta,\ \ |V_{CD}| = IR_0 \cos \omega\delta,$$

and

$$V_0 = |V_{AB}| - |V_{CD}| = \sqrt{2}\, IR_0 \sin (\omega\delta - \pi/4).$$

Figure 3(*b*) :

$$|V_{AB}| = E \cos \omega\delta,\ \ |V_{CD}| = E \sin \omega\delta,$$

$$V_0 = |V_{AB}| - |V_{CD}| = \sqrt{2}\, E \sin (\omega\delta - \pi/4).$$

In figure 3(b) the input impedance is R_0; the equivalent current fed form is as shown in figure 3(c).

Figure 3(*c*) :

$$|V_{AB}| - |V_{CB}| = \sqrt{2}\, R_0 I \sin (\omega\delta - \pi/4).$$

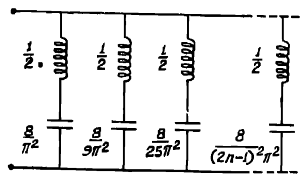
Figure 3(*d*) :

$$|V_{BA}| = \frac{V}{2}(1 - \cos 2\omega\delta),\ |V_{CD}| = \frac{V}{2}(1 + \cos 2\omega\delta),$$

$$|V_{CA}| = V \sin \omega\delta,\qquad |V_{FC}| = V \cos \omega\delta.$$

The arrangements described in the foregoing discussion are physically realisable at u.h.f. as well as in the r.f. and lower frequency ranges.

It is also to be noted that in the arrangements of figure 1 and figure 2, both voltage fed and current fed systems are equally practicable. Further, in the system of figure 2, one can employ lumped equivalents of the input impedance of open- or short-circuited transmission lines, some of which are shown in figure 4. Now



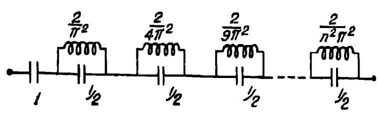


Fig. 4. Lumped equivalent of the input impedance of an open ended transmission line. The capacitances are in units of $\frac{\delta}{R_0}$ and the inductances are in units of δR_0.

for operation at higher frequencies, δ and hence R_0 have necessarily to be small, thus requiring the use of current fed forms. If high impedance cables are available, one may use the voltage fed forms.

Variation of the centre frequency : A variation of the centre frequency can be effected by adjusting the relative magnitudes of the two voltages (i.e., A_1 or A_2). It is observed from Eq.(4) that a variation of the amplitude factor in the cosine component by 50% results in a variation of the centre frequency by 25%. This, however, involves a change in the difference output.

EXPERIMENTAL ARRANGEMENT

An experimental arrangement as shown in figure 5 was made. The value of R chosen was one that gave a square pulse with step input to the system with least reflections. The characteristic impedance was 950 Ω and the total delay (one way) 0.28μ secs. The output impedance of the cathode follower was about 200Ω. The voltage across the resistance R was consequently reduced by the factor $\frac{15}{19}$ and the centre frequency shifted to a value greater than $\frac{1}{8\delta}$. The latter could, however, be moved back by adjusting the potentiometer shown in figure 5. The system was tested by applying C. W. signal from a standard signal generator and observing the detected outputs $|V_{AB}|$ and $|V_{BG}|$. It is found from figure 6 that the detected difference output is substantially linear over the frequency range 200 kc/s to 600 kc/s, the centre frequency in this case being 400 kc/s.

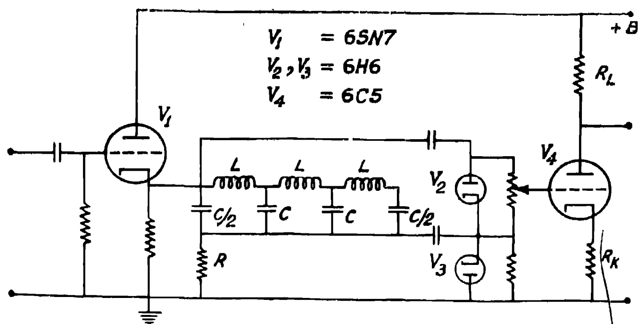


Fig. 5. Circuit arrangement of a transmission line discriminator.

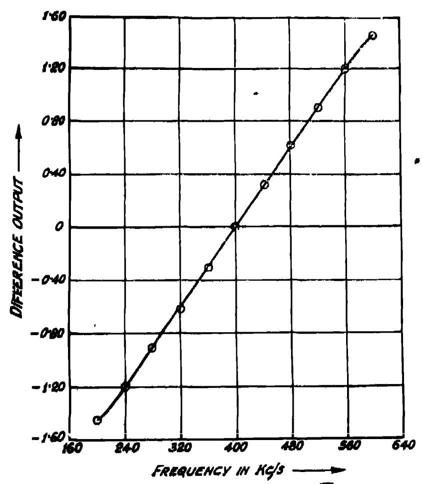


Fig. 6. Measured characteristics of the system of figure 5.

Another arrangement corresponding to the system in figure 3(c) was also made as shown in figure 7 and tested. The results obtained are presented in figure 8 which shows that almost perfect linearity extends up to ±3mc/s round the centre frequency of 6.4 mc/s.

It is thus seen that in both the discriminator arrangements the output is quite linear up to a frequency deviation of about 50% of the centre frequency.

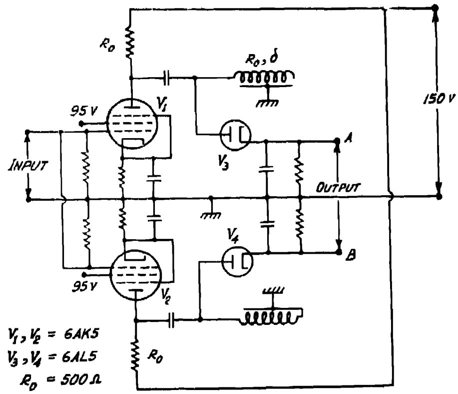


Fig. 7. Alternative circuit arrangement of a transmission line discriminator.

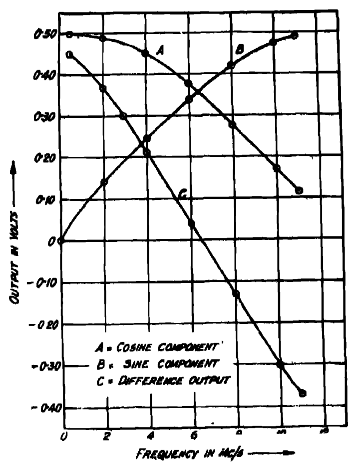


Fig. 8. Measured characteristics of the system of figure 7.

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